DISCRETE EVENT CONTROLLER SYNTHESIS FOR FORCE-CONTROLLED ASSEMBLY TASKS WITH FRICTION

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ABSTRACT

An existing discrete event controller synthesis methodology for force-controlled systems is extended to include friction. Friction poses many problems if a rigid body model is used, including cases where no solutions or multiple solutions exist. Instead, we use a compliant body model. This model permits the analytical synthesis of force control command constraints for each discrete state in the model, including multi-contact situations and accounting for frictional forces. These constraints are then solved to determine a force control command. The applicability and effectiveness of the method are experimentally demonstrated with a peg-in-hole assembly task.

KEYWORDS: *discrete event control, force control, assembly, friction*

INTRODUCTION

Position errors have been shown to be the primary cause of failure in robotic assembly [10]. Instead, force control is a natural paradigm for assembly tasks as it is fundamentally compliant, virtually eliminating any dependence on position. A discrete event controller synthesis technique for force-controlled systems has been developed [2, 1]. However, these previous works did not consider friction. When using force control, friction is a major consideration and must be included to avoid jamming and wedging. This paper presents a discrete event controller synthesis technique for force-controlled systems with friction.

During an assembly task, the dynamics of the system change in a discrete fashion as the workpiece becomes constrained by the environment. Most research into assembly tasks is conducted in a continuous-time framework and neglects the discrete event

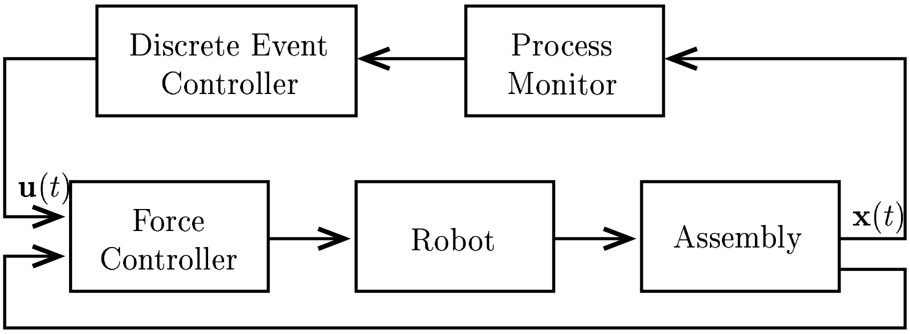


Figure 1: *Block Diagram Representation of System*

nature of these systems. Discrete event modelling allows the consideration of both the continuous-time system and the discrete events that occur when a constraint is encountered. There have been many wide-ranging applications of discrete event systems including network protocols [11] and manufacturing systems [4]. Stiver and Antsaklis [13] gave a general formulation for the modelling and analysis of discrete event systems. In particular, assembly tasks have been controlled using discrete event systems by Moody *et al.* [9] and McCarragher [8]. Also, Tarn *et al.* [14] considers the dynamics of gain and loss of contact in depth.

Work in the field of force control tends to concentrate on low-level details of the control task, and as such has had limited success despite significant effort. In 1985, Hogan [5] developed impedance control for the manipulation of objects constrained by the environment and since then, a great deal of research has been conducted into a number of variants of force control.

In prior works [1, 2] the authors presented a discrete event controller synthesis methodology for force-controlled systems, neglecting friction. Friction poses many problems if a rigid body model is used [7, 12], including cases where no solutions or multiple solutions exist. For this paper, we use a compliant body model which permits the analytical synthesis of force control commands which account for friction. We will model the assembly process as the discrete event system as shown in Figure 1. For the purposes of this paper, the control command $\mathbf{u}(t)$ will be a force/torque vector or a *wrench*:

$$\mathbf{u}(t) = \begin{bmatrix} \mathbf{F} \\ \mathbf{T} \end{bmatrix} \tag{1}$$

where $\mathbf{F}$ is the force and $\mathbf{T}$ the torque.

FORCE CONTROL CONDITIONS

Control commands are determined by establishing a *desired discrete event* for each discrete state. The desired event is chosen such that the system moves towards the completion of the assembly. The series of desired events gives a discrete event trajectory which accomplishes the assembly task. For any given discrete state, we use the desired event to establish three conditions on the contact forces, as follows.

Maintaining Condition

The first possible requirement of the force/torque control command is that it maintains existing contacts. To achieve this the normal component of the contact force must be positive.

$$\mathbf{F}_i \ . \ \mathbf{n}_i > 0 \tag{2}$$

where $\mathbf{F}_i$ is the contact force for the i^{th} edge-edge or surface-vertex contact and $\mathbf{n}_i$ is the unit normal. Equation (2) is our maintaining condition and is used to maintain an existing contact.

Enabling Condition

In addition to maintaining the existing constraint, we must consider the forces required for the system move to the next discrete state. For a gain of contact we require that the normal component of the contact force becomes positive.

$$\mathbf{F}_j \ . \ \mathbf{n}_j > 0 \tag{3}$$

Similarly, for a loss of contact, we require that the normal component of the contact force becomes zero.

$$\mathbf{F}_j \ . \ \mathbf{n}_j = 0 \tag{4}$$

Hence, equations (3) and (4) are conditions used to enable the next transition. Equation (3) is used for a gain of contact and equation (4) is used to lose an existing contact.

Disabling Condition

The third condition, the disabling condition, is used to prevent unwanted gains of contact. This condition is derived directly from the enabling condition for a gain of contact. If equation (3) is necessary for a gain of contact then the following condition is sufficient to prevent a gain of contact.

$$\mathbf{F}_k \ . \ \mathbf{n}_k = 0 \tag{5}$$

The desired event determines which of the above conditions should be applied for each possible contact. The maintaining condition is (2) and is used when it is desired to maintain a contact. The enabling conditions are (3) and (4) and are used to enable the gain or loss of a contact, respectively. The disabling condition is (5) and is used to prevent unwanted gains of contact. However, these are conditions upon the contact forces at the various contact points. The following section will develop a relationship between the control command and the contact forces.

FORCE COMMAND CONSTRAINTS

The previous section developed constraints upon the contact forces to maintain a contact, and to enable or disable a contact change. However, the force control command is applied at a single point on the workpiece and we do not have direct control over the contact forces. Hence, we must relate the contact forces to the force control command. The relationship between the contact forces and the control command depends upon the assumed behaviour of the bodies in the system and the model used for the frictional forces.

Contact Force Model

Many authors have traditionally used a rigid body model with Coulombic friction. Unfortunately, this model has a number of drawbacks including simple cases with no consistent solutions and many cases with multiple possible solutions [7, 15]. For example, Mason and Wang [7] discus the case of a thin cylindrical rod sliding along a planar surface. In most cases, the contact forces balance the gravitational force, so that the end of the rod either moves sideways or accelerates away from the surface. However, for a particular choice of parameters, a rigid body model leads to an angular acceleration which drives the end of the rod into the surface. This solution is inconsistent with the rigid body model.

A compliant body model offers an alternative which does not suffer from these inconsistencies [6, 15]. A compliant body model assumes that contacts cause deformations of the bodies involved. The deformation is modelled as a spring and the unique solution is determined by minimising the energies stored in the springs. In addition, a basic compliant body model, neglecting dynamic effects, results in a piecewise linear relationship between displacement and force. We will exploit this linear relationship in the analysis of multi-contact situations. For a full discussion of the compliant body model used, see [3].

Single-Point Contact Forces

Consider the forces on the workpiece in the single-contact situation as presented in Figure 2. The only forces applied to the workpiece are a wrench applied at point O and a contact force resulting from the interaction with the environment. Here we include frictional forces, as indicated by the tangential component of $\mathbf{F}_i$ in Figure 2. If the applied wrench is given by equation (1) then the equations of motion of the workpiece may be written as

$$\mathbf{F} + \mathbf{F}_i = \mathbf{Ma} \tag{6}$$

$$\mathbf{T} + \mathbf{r}_i \times \mathbf{F}_i = \mathbf{I}_o \alpha \tag{7}$$

where $\mathbf{M}$ is the mass matrix for the workpiece, $\mathbf{a}$ is the acceleration vector, $\mathbf{I}_o$ is the inertia matrix about point O (see Figure 2) and α is the angular acceleration of the workpiece about point O. In most cases the mass of the workpiece and the accelerations involved will be small enough that $\mathbf{Ma} \approx 0$ and $\mathbf{I}_o \alpha \approx 0$. However, the effects of the mass and moment of inertia may be accounted for by writing

$$\mathbf{F}' = \mathbf{Ma} - \mathbf{F} \tag{8}$$

$$\mathbf{T}' = \mathbf{I}_o \alpha - \mathbf{T} \tag{9}$$

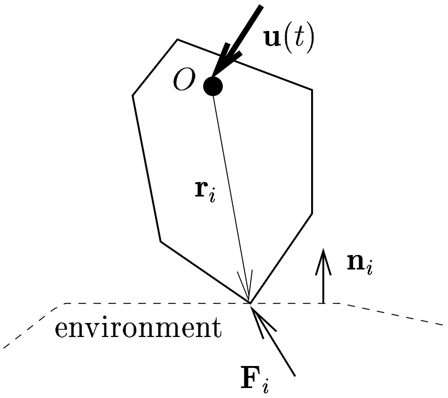


Figure 2: Forces Applied to the Workpiece in a Single-Contact Situation

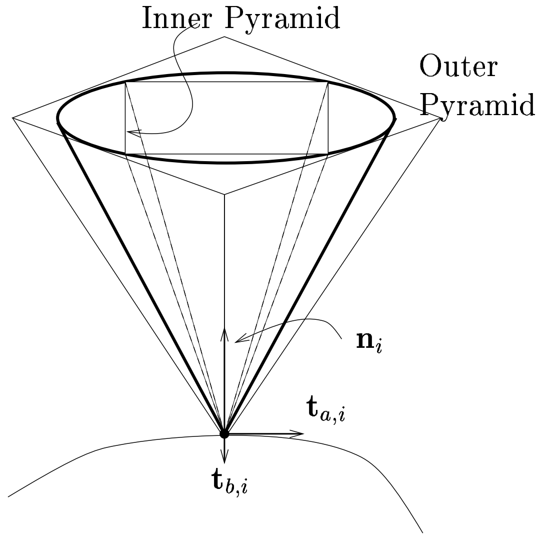


Figure 3: Friction Cone and Pyramid Approximation

and finding conditions on $\mathbf{F}'$ and $\mathbf{T}'$. We substitute into (6) and (7) to give

$$\mathbf{F}' = \mathbf{F}_i \tag{10}$$

$$\mathbf{T}' = \mathbf{r}_i \times \mathbf{F}_i \tag{11}$$

We define

$$\hat{\mathbf{T}}_i = \mathbf{r}_i \times \mathbf{n}_i \tag{12}$$

where $\hat{\mathbf{T}}_i$ is the minimum torque vector which results in a unit normal contact force. Conditions relating the normal contact force to the applied wrench were derived in detail in [2]. The condition for a zero normal contact force is:

$$\mathbf{F}_i \ . \ \mathbf{n}_i = 0 \Leftrightarrow \begin{bmatrix} \mathbf{n}_i & \frac{\hat{\mathbf{T}}_i}{|\hat{\mathbf{T}}_i|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} = 0 \tag{13}$$

Similarly, the condition for a positive contact force is:

$$\mathbf{F}_i \ . \ \mathbf{n}_i > 0 \Leftrightarrow \begin{bmatrix} \mathbf{n}_i & \frac{\hat{\mathbf{T}}_i}{|\hat{\mathbf{T}}_i|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} > 0 \tag{14}$$

Using equations (14) and (13) we may rewrite the maintaining, enabling and disabling conditions of equations (2), (3), (4) and (5) as constraints upon the force control command. The maintaining constraint is then:

$$\begin{bmatrix} \mathbf{n}_i & \frac{\hat{\mathbf{T}}_i}{|\hat{\mathbf{T}}_i|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} > 0 \tag{15}$$

Constraints for the enabling and disabling conditions are derived in a similar manner.

Slip Conditions

To ensure that the principle of superposition applies, we must derive additional conditions: one to prevent a static contact from slipping and a second to prevent a slipping contact from sticking. Unfortunately, a minor restriction is required: upper and lower bounds must be established upon the normal force.

$$0 < F_{min} < |\mathbf{F}_i| < F_{max} \tag{16}$$

Intuitively, the lower bound on the normal force allows the specification of a permissible range of tangential forces for which the contact sticks. Similarly, the upper bound on the normal force gives a lower bound on the force required to cause sliding. The selection of the upper and lower bounds depends upon the application.

Now we can prevent slip by ensuring that

$$|\mathbf{F}_{t,i}| < \mu F_{min} \tag{17}$$

and we can ensure slip by requiring

$$|\mathbf{F}_{t,i}| > \mu F_{max} \tag{18}$$

where μ is the coefficient of friction.

Unfortunately, these conditions are difficult to use as they are bounds on the magnitude of the tangential force vector. As the bounds are based on a simple friction model and the coefficient of friction is usually poorly known, a simpler approximation is justified. Hence, we will approximate the friction cone by pyramids as shown in Figure 3. The pyramid that is circumscribed by the friction cone is used to constrain the tangential forces to prevent slip and the outer pyramid is used to ensure slip. These pyramids were selected to simplify the resultant constraints for slipping and sticking.

First we write expressions similar to equation (12) for the tangential directions

$$\hat{\mathbf{T}}_{a,i} = \mathbf{r}_i \times \mathbf{t}_{a,i} \tag{19}$$

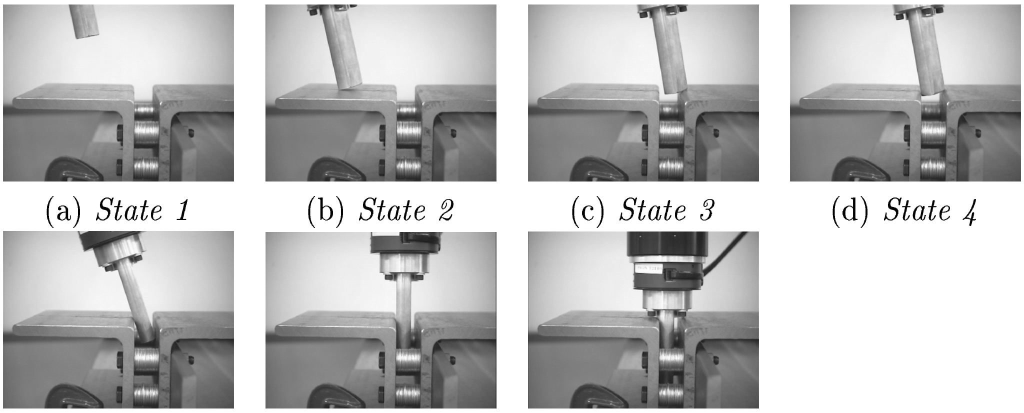
$$\hat{\mathbf{T}}_{b,i} = \mathbf{r}_i \times \mathbf{t}_{b,i} \tag{20}$$

and now we may write constraints upon $\mathbf{F}'$ and $\mathbf{T}'$. To ensure no slip, we want to limit the magnitude of the forces parallel to the axes $\mathbf{t}_{a,i}$ and $\mathbf{t}_{b,i}$. Using a method similar to that derived for the normal force results in the following conditions to prevent slip

$$\left| \begin{bmatrix} \mathbf{t}_{a,i} & \frac{\hat{\mathbf{T}}_{a,i}}{|\hat{\mathbf{T}}_{a,i}|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} \right| < \frac{1}{\sqrt{2}} \mu F_{min} \tag{21}$$

and

$$\left| \begin{bmatrix} \mathbf{t}_{b,i} & \frac{\hat{\mathbf{T}}_{b,i}}{|\hat{\mathbf{T}}_{b,i}|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} \right| < \frac{1}{\sqrt{2}} \mu F_{min} \tag{22}$$



(a) *State 1* (b) *State 2* (c) *State 3* (d) *State 4*

(e) *State 5* (f) *State 6* (g) *State 7*

Figure 4: *States of the Experimental Assembly Task*

To ensure slip, we require that the tangential force is greater than the maximum sustainable frictional force. Using the outer pyramid approximation of Figure 3 results in the following condition to ensure slip

$$\left\| \begin{bmatrix} \mathbf{t}_{a,i}+\mathbf{t}_{b,i} & \frac{\hat{\mathbf{T}}_{a,i}}{\left|\hat{\mathbf{T}}_{a,i}\right|^2}+\frac{\hat{\mathbf{T}}_{b,i}}{\left|\hat{\mathbf{T}}_{b,i}\right|^2} \end{bmatrix} \begin{bmatrix} \mathbf{F}' \\ \mathbf{T}' \end{bmatrix} \right\| > \sqrt{2}\mu F_{max} \qquad (23)$$

So equations (21) and (22) are used to ensure that slip does not occur for a contact that must remain static and equation (23) is used to cause a contact to slip.

EXPERIMENTAL RESULTS

The controller synthesis technique presented above was applied to a 4 degree-of-freedom assembly task as shown in Figure 5. In this system the robot manipulator is controlled by an impedance controller which in turn is controlled by force commands from the discrete event controller. The force commands are re-computed 5 times a second to allow for changes in the position and orientation of the peg. The initial orientation is such that the peg must rotate about its long axis before insertion. Hence, this task requires four degrees of freedom and requires a general controller capable of dealing with spatial tasks, rather than a restricted planar controller. To complete the assembly task, the controller follows the discrete state trajectory illustrated in Figure 4.

Previous implementations of this insertion task were prone to jamming and wedging as they neglected frictional forces. For example, in the second state, the robot must slide the peg across the surface, whilst maintaining contact. Figure 5 shows the difference between the control commands determined in [2] and the control command used here, which considers friction. It is clear that the command $\mathbf{F}_f$ is much more suitable for sliding across the surface.

In addition to the ability to avoid jamming and wedging, this implementation of the discrete event controller proved highly robust and capable of completing the

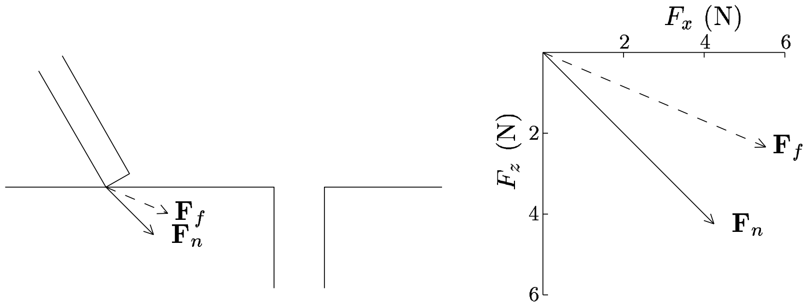


Figure 5: *Comparison of force commands:* $\mathbf{F}_n$ *is the force command neglecting friction whilst* $\mathbf{F}_f$ *compensates for friction using equation (23)*

assembly task with positioning errors of up to $50mm$ in any direction and orientation errors of up to 30° about the vertical axis.

CONCLUSIONS

A discrete event controller synthesis technique for the successful control of force-controlled constrained motion systems with friction was presented. This paper significantly extends previous work by considering friction when synthesising the force control command. For a force-controlled system, friction is a very important aspect and must be considered to avoid wedging and jamming. In this paper, additional constraints were added to compensate for the frictional forces. The experimental results clearly demonstrate that the new controller successfully accounts for friction and avoids wedging and jamming. The controller synthesis technique presented here is a ideal tool for assembly as in combines the compliance of force control with the excellent error-recovery characteristics of discrete event control.

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